Quantitative Analyze Report about A Structured Product

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1.BS Model

{a}

volatility=17.80%, Rate at 2024-01-01 00:00:00=4.01%

Yearly volatility would be the standard deviation of log return multiply with square root of trading days in one year.

{b}

g=0.00%, G=7.27%

g=0.50%, G=6.92%

g=1.00%, G=6.57%

g=1.50%, G=6.20%

This contract could be regarded as a combination of a forward contract, a long position in a call option with lower strike and a short position in a call option with higher strike, or, a forward contract and a bull spread.

A superclass BullSpread is defined, which contains some most basic properties of this contract (Stock price, g, G, interest rate, contract start time, contract end time, volatility, and deposit). In the class, I wrote some methods to calculate call option price under BS model and forward price.

We could set an equation, the analytical price of contract would be equal to the deposit.

Given a list of g, we could build an object in BullSpread class by given information. Then by invoking the functions in BullSpread Class, we could use Newton’s method to calculate corresponding G.

{c}

Rate at 2021-01-01 00:00:00: 0.16%

g=0.00%, G=0.36%

Delta hedged results=$-665.91

Risk-free rate at 01/01/2021 is downloaded to calculate the new g-G pair.

Another method under BullSpread is defined, initialize the Delta, Fund account(Before Rebalance), Bank account(After Rebalance) list. I set a loop to calculate the movements of Fund account, the contract price according to new information, delta of bull spread, and Bank account according to stock price chart. The method would return the gap between final Fund account and payoff.

By invoking the Delta Hedge method, the return value would be the P&L and hedged portfolio.

2.Vasicek Model

{a}

A subclass VasicekBullSpread is defined, extending the BullSpread superclass, and some new properties are added into the class (alpha, beta, gamma, rho). In the class, I wrote some methods to calculate call option price under Vasicek model and forward price.

Set the guess value of alpha, beta gamma and rho.

Similarly, we could set an equation as done before.

Given a list of g, we could build an object in VasicekBullSpread class by given information. Then by invoking the functions in VasicekBullSpread Class, we could use Newton’s method to calculate corresponding G.

{b}

Estimated Parameters:

Alpha:0.19888

Beta:0.03354

Gamma:0.01035

Rho:0.20813

g=0.00%, G=8.64%

g=0.50%, G=8.09%

g=1.00%, G=7.55%

g=1.50%, G=7.00%

g=2.00%, G=6.45%

A maximum likelihood estimation function is defined to optimize alpha, beta and gamma. In the function, I apply the Vasicek process to calculate the likelihood of the model. SciPy. Optimization libraries would help us get the best fit parameters’ estimates.

I wrote 2 functions to simulate the movements of interest rate and stock price by given optimized alpha and beta. Rho could be estimated by the correlation of the 2 simulated series.

Applying calibrated alpha, beta, gamma and rho, we could again calculate the g-G pairs.

{c}

Rate at 2021-01-01 00:00:00: 0.16%

g=0.00%, G=0.32%  
Rho hedged results=$-267.36

Delta-Rho hedged results=$122.95

Similarly, risk-free rate at 01/01/2021 is applied to calculate the new g-G pair.

Delta-Rho Hedge would be very similar to what we have done in Delta Hedge part under BS model. But we need to calculate the sensitivity of interest rate hedging instrument to contract by calculating the sensitivity of hedging instrument to forward, call option with lower strike and call option with higher strike separately (rho-1, rho--2, rho-3). In the rho hedge method, I dynamically updated the (rho-1, rho-2, rho-3) as well as movements of Fund account, the contract price and delta, and Bank account.

By invoking the Delta-Rho Hedge method, the return value would be the P&L and Delta-Rho hedged portfolio.

3.Stochastic Volatility-Heston Model

{a}

Define a model with Vasicek Process interest rate and Heston Process volatility:

Where initial parameters should be:

Kappa:10.00000

Theta:0.03514

Volvol:0.42425

Rho1:-0.71006

Alpha:0.19888

Beta:0.03354

Gamma:0.01035

Rho2:0.20813

I downloaded the time series data VIX index, which is needed to calculate theta, volatility of volatility and rho.

A variable X2t is set to be the difference between . Write a function with return value X2t. Define another function would give the average of X2t. The return of average X2t is . Use the to simulate a series of X2t. X2t should be a martingale with distribution N(0,).

Another variable Y2t is set to be the difference between St. Write a function of mean of Y2t and solving optimization problems to get , and use to simulate Y2t process. We estimated by the correlation of X2t and Y2t.

In this case, we get our initial parameters of stochastic volatility process.

{b}

Optimized Parameters:

Kappa:13.61236

Theta:0.03820

Volvol:0.22239

Rho1:-0.71006

Alpha:0.19888

Beta:0.03354

Gamma:0.01035

Rho2:0.20813

I applied the same way optimizing parameters under Vasicek Model here. Another maximum likelihood estimation function is defined to optimize kappa, theta and volatility of volatility. In the function, I apply Heston process to calculate the likelihood of the model.

{c}

g=0.00%, G=9.61%

g=0.50%, G=8.94%

g=1.00%, G=8.26%

g=1.50%, G=7.58%

g=2.00%, G=6.90%

Again, a subclass StochasticVolatilityBullSpread is defined, extending the VasicekBullSpread superclass, and some new properties are added into the class (volatility at start point, kappa, theta, volatility of volatility, rho). In the class, I wrote some methods to calculate call option price under this special model and forward price. Instead of analytical approaches, I applied Monte Carlo methods to simulate stochastic volatility and interest rate process here. Given a list of g, we could build an object in StochasticVolatilityBullSpread class by given information. Then we could use Newton’s method to calculate corresponding G.

Compare with BSM and Vasicek (g-G gap-BS<Vasicek <Stochastic Volatility):

Under Black Scholes Model and Vasicek Model, we assume that the volatility is at historical average volatility level, which is 17.80%. But in Heston Model, we start with the implied volatility of SP500 at 01/01/2024, which is 13.2%, and will asymptotically back to mean reversion level. we can posit that it would be for a long time keep below the mean level. Usually, when volatility rises the option premium also increases. If we postulate the same g-G, the price of contract based on Black-Scholes and Vasicek (Higher Volatility) would be higher than which based on Stochastic Volatility Process (Lower Volatility). In order to keep the contract price constant, we should increase the gap between g-G based on Stochastic Volatility Model.

4.Machine Learning

{a}

Pricing:

1. Feature Recognition: Identify and extract relevant features from the data, like trends, volatility, and correlations.
2. Model Training: Use machine learning algorithms to train a model on the historical data. The model learns patterns and relationships between features and asset prices.
3. Validation: Assess the model's performance using validation data to ensure it accurately captures market dynamics.
4. Prediction: Once validated, the model can be used to predict future asset prices based on current market conditions and feature inputs.

Hedging:

1. Risk Factor Identification: Analyze historical market data identify key risk factors that influence the portfolio's value, such as asset prices, interest rates, and economic indicators.
2. Model Training: Train a machine learning model to develop hedging strategies that mitigate portfolio risks while maximizing returns.
3. Dynamic Hedging: Continuously monitor market data and risk factors to dynamically adjust portfolio positions and hedging strategies in response to changing market conditions.
4. Evaluation and Optimization: Evaluate the effectiveness of hedging strategies and adjust the model iteratively to optimize performance and adapt to evolving market dynamics.

{b}

1. Complexity Handling: ML techniques can handle complex relationships among factors affecting financial instrument prices better than traditional methods. They're good at dealing with high-dimensional and non-linear data.
2. Flexibility: ML models are adaptable to changing market conditions and data characteristics. They can easily integrate new data and improve over time without needing manual adjustments.
3. Feature Recognition: ML automatically identifies relevant features from raw data, reducing the need for manual feature recognition. In complex financial markets where many factors influence prices and risks and hard for humans to identify by analytical way.
4. Trading Cost: ML models can account for trading costs explicitly, unlike traditional methods that often assume bid and ask prices are the same. This improves the accuracy of pricing and hedging strategies.

{c}

CHALLENGES:

1. Data Resources: Financial data can be messy and incomplete, making it tough to ensure accuracy and consistency for training ML models.
2. Guarding Against Overfitting: Models can get too caught up in past data, making them less reliable for future predictions. Ensuring they adapt well to new situations is key for solid strategies.
3. Understanding Predictions: ML models often operate like black boxes, making it hard to grasp why they make certain predictions. Techniques that shed light on their decisions are crucial for trust and compliance.
4. Computational Demands: Running ML models requires serious computing power and infrastructure. Expanding ML capabilities means investing in technology and resources.

SET UP MEASURES:

1. Data Management: Strong systems needed to handle lots of financial data from different sources.
2. Model Building and Testing: Thorough processes for creating and testing ML models, including preparing data, picking features, and checking performance.
3. System Integration: ML pricing and hedging tools need to work smoothly with existing trading and risk management systems.
4. Continuous Oversight: Keeping an eye on ML models, tweaking them as needed to stay accurate and compliant with market rules.